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Odd–parity negative modes of Einstein–Yang–Mills black holes and sphalerons

Mikhail S. Volkov[1]

Institut für Theoretische Physik der Universität Zürich–Irchel,
Winterthurerstrasse 190, CH–8057 Zürich, Switzerland,
e–mail: volkov@physik.unizh.ch

Dmitri V. Gal'tsov[2]

Centro de Investigación y de Estudios Avanzados del I.P.N.,
Departamento de Física, Apdo. Postal 14-740, 07000, México, D.F., México,
e–mail: dgaltsov@fis.cinvestav.mx

Abstract

An analytical proof of the existence of negative modes in the odd–parity perturbation sector is given for all known non-Abelian Einstein–Yang–Mills black holes. The significance of the normalizability condition in the functional stability analysis is emphasized. The role of the odd–parity negative modes in the sphaleron interpretation of the Bartnik–McKinnon solutions is discussed.

[1]On leave from Physical–Technical Institute of the Academy of Sciences of Russia, Kazan 420029, Russia

[2]On leave from the Dept. of Theoretical Physics, Moscow State University, 119899 Moscow, Russia; e–mail: galtsov@grg.phys.msu.su

1 Introduction

Soon after the discovery of regular particle–like [1] and black hole [2] solutions in the $SU(2)$ Einstein–Yang–Mills (EYM) theory, their instability was demonstrated [3]. Numerical analysis has revealed the existence of n (the number of nodes of the YM function) negative modes in the *even–parity* spherical perturbation sector; moreover, the investigation of the non-linear dynamics of perturbations has clearly shown the instability [4] (an interesting global analysis can also be found in [5]). Some time later, a generalization of the $SU(2)$ black hole solutions for the non-vanishing charge was found in the EYM theory for a larger gauge group [6], however, the issue of stability for these solutions has been open so far.

It is worth noting that, for the regular Bartnik–McKinnon (BK) solutions, there exists at least one negative mode in the *odd–parity* sector too. This mode has the same nature as negative mode of the electroweak sphaleron solution [7], [8], indicating the presence of a potential barrier separating topologically distinct YM vacua. This plays the crucial role in the proposed sphaleron interpretation for the BK solutions [10], and the related physical issues [11]. The existence of the odd–parity negative mode has been shown analytically using variational techniques [10]. The idea consists in the construction of an energy–reducing function sequence on the constraint surface in configuration space of the theory. The advantage of this technique is that it does not require the detailed knowledge of the background equilibrium solution. Recently this approach has been successfully applied in the analysis of stability of regular solutions of the EYM–Higgs theory [12], and regular solutions of the EYM theory for an arbitrary gauge group [13]. The latter result shows, in particular, the instability of the non-trivial $SU(3)$ EYM regular solutions found recently by Künzle [14].

For EYM black holes, the variational approach can be applied as well. For the $SU(2)$ solutions, a sequence of trial functions reducing the total ADM energy has been constructed in [15]. More detailed analysis reveals, however, that the decrease of the energy is not sufficient to demonstrate instability. Another important condition is necessary: the energy reducing perturbations must be normalizable. Unfortunately, for the trial functions used in our earlier paper [15], the normalizability condition was not satisfied because of their inappropriate behaviour at the event horizon (for the regular solutions such a problem does not arise). Surprisingly enough, we find that a similar situation can be often met in other papers, where the argument based on the existence of the energy reducing fluctuations is used — little attention is usually paid to the normalizability condition.

The purpose of this paper is two–fold: first, we want to elucidate the nature and to emphasize the importance of the normalizability condition for the general field–theoretical stability analysis. Secondly, we investigate the structure of the *odd–parity* spherical perturbation sector for the static EYM black hole and sphaleron solutions. We shall give a proof of the existence of negative modes in this sector for all known essentially non-Abelian EYM black holes, and our proof is valid in the regular case as well. This justifies the main statement made in [15] concerning the $SU(2)$ EYM black holes. In addition, we analyse stability of magnetic $U(1)$ black holes within the context of EYM theory, and discuss the importance of the odd–parity negative modes for understading the physical nature of the localized finite energy EYM solutions.

2 The functional criteria of instability

Consider a theory of a classical field, or several fields, denoted commonly by ϕ. Assume that the theory admits a localized static equilibrium solution $\phi_s(\mathbf{x})$ possessing finite energy $E[\phi_s(\mathbf{x})]$. Suppose one has found a sequence of static field configurations, $\phi_\lambda(\mathbf{x})$, such that $\phi_{\lambda=0}(\mathbf{x}) = \phi_s(\mathbf{x})$, and the energy has a maximum for $\lambda = 0$:

$$E[\phi_{\lambda\neq0}(\mathbf{x})] < E[\phi_s(\mathbf{x})]. \tag{1}$$

The mechanical picture for this situation is that of a particle sitting at the top of potential hill whose profile is given by $U(\lambda) = E[\phi_\lambda(\mathbf{x})]$. The question arises whether (1) is sufficient in order to reveal an instability of the solution $\phi_s(\mathbf{x})$. Strictly speaking, the answer is negative. Namely, the equilibrium state of the "particle" will be unstable only if its effective mass is finite. Otherwise, the particle can not depart from the top due to infinite inertia.

Let us reformulate this as follows. If one allows for a time evolution along the sequence $\phi_\lambda(\mathbf{x})$, replacing the parameter λ by a function $\lambda(t)$, and if one finds that the kinetic energy, i.e. the part of the total energy proportional to $\dot\lambda^2$, is finite, then (1) implies indeed an instability. Otherwise, $\phi_\lambda(\mathbf{x})$ is not a physically acceptable sequence, and nothing can be inferred from Eq.(1).

To put this into more rigorous form, consider small fluctuations $\delta\phi(t, \mathbf{x})$ around the static equilibrium solution. Linearizing the field equations and specifying the time dependence as $\delta\phi(t, \mathbf{x}) = \exp(-i\omega t)\Psi(\mathbf{x})$ one can usually represent the perturbation equations as [12]

$$H\Psi = \omega^2 M\Psi. \tag{2}$$

Here, the two operators H and M depending on $\phi_s(\mathbf{x})$ are assumed to be independent on ω. H is usually self-adjoint with respect to a properly defined scalar product $\langle\Psi|\Phi\rangle$, and the kinetic energy matrix M is positive definite, $\langle\Psi|M|\Psi\rangle > 0$.

Clearly, the equilibrium static solution will be unstable if a normalizable solution to (2) with $\omega^2 < 0$ can be found. However, this may require numerical calculations. The simpler (though less informative) way to reveal an instability is to make use of the minimum principle for the following functional defined through (2):

$$\omega^2(\Psi) = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|M|\Psi\rangle}, \tag{3}$$

with Ψ being a trial function. The lowest eigenvalue is known to correspond to the lower bound for this functional. From here it follows that if a Ψ can be found such that $\omega^2(\Psi) < 0$, then the operator H is not positive definite, and negative eigenvalues therefore do exist.

Obviously, $\omega^2(\Psi) < 0$ implies that

$$\langle\Psi|H|\Psi\rangle\ < 0, \tag{4}$$

which is equivalent to (1). This condition is frequently used in order to demonstrate instability in field–theoretical systems (see for instance [7], [9]). However, for such systems the operator M is generally unbounded. Often, the trial functions from the domain of H, while ensuring (4), lead to *divergence* of the expectation value $\langle\Psi|M|\Psi\rangle$. In this case

(4) is insufficient to establish instability. We therefore arrive at the second important condition

$$\langle \Psi | M | \Psi \rangle \ < \infty. \tag{5}$$

Physically this can be seen as the condition assuring the finiteness of the "kinetic energy" associated with time–dependent perturbations.

We deliberately spend so much time on these subtleties since in the existing literature they seem often to be overlooked. Sometimes the trial functions used automatically fulfill condition (5), but not always. One may think that condition (5) is somehow implied by (4), in the sense that, if a function subject to (4) is known, then one can in principle find another function satisfying both (4) and (5). There are, however, examples when condition (4) holds even for perfectly stable solutions — but not both (4) and (5). Consider, for instance, the Bogomolny–Prasad–Sommerfield monopole solutions [16]. They form a family $\phi_{BPS}(\mathbf{x}, v)$ whose members are distinguished by the asymptotic value of the Higgs field, v, and satisfy the same system of equations; the corresponding mass is proportional to v. Given a member of the family specified by some v_0, one can construct the trial sequence $\phi_\lambda(\mathbf{x}) = \phi_{BPS}(\mathbf{x}, v_0 - \lambda^2)$ which reduces the energy. However, the norm (5) is divergent due to the change of the asymptotic values of the fields.

Finally, we sketch an idea of the rigorous justification of the above functional criteria for instability. Assume that the matrix M in (2) is non-degenerate and symmetric. Perform a linear transformation $\Psi = O\tilde{\Psi}$ such that $O^T M O = 1$ and the new Hamiltonian $\tilde{H} = O^T H O$. Omitting tilde, one can represent (2) as an eigenvalue problem, $H\Psi = \omega^2 \Psi$, for the unbounded (but usually bounded from below) operator H acting on the Hilbert space $\mathcal{H}$ of Ψ with a scalar product $\langle \Psi | \Phi \rangle = \int \Psi^\dagger \Phi \, d\mu(\mathbf{x})$, where $d\mu$ is an appropriately chosen measure. Usually, one can easily specify a dense set $\mathcal{D}(H) \subset \mathcal{H}$ as the domain of H in such a way that H is symmetric on $\mathcal{D}(H)$. Next, assume that *the existence* of a self-adjoint extension H_1 is known, $\mathcal{D}(H_1) \supset \mathcal{D}(H)$. Then, the spectral decomposition for H_1 implies the following inequality for the ground state eigenvalue:

$$\omega_0^2 \leq \omega^2(\Psi), \quad \text{where} \quad \omega^2(\Psi) = \frac{\langle \Psi | H_1 | \Psi \rangle}{\langle \Psi | \Psi \rangle}, \quad \Psi \in \mathcal{D}(H_1). \tag{6}$$

Thus, if $\omega^2(\Psi) < 0$ for some $\Psi \in \mathcal{D}(H_1)$, then $\omega_0^2 < 0$. Notice that if one chooses $\Psi \in \mathcal{D}(H) \subset \mathcal{D}(H_1)$ then one can replace H_1 by H in the definition of $\omega^2(\Psi)$. The condition $\omega^2(\Psi) < 0$ is then equivalent to the following two conditions. The first one is given by (4), where the trial function Ψ has to be an element of the already specified set $\mathcal{D}(H) \subset \mathcal{H}$. The second, the normalizability condition, becomes now fairly trivial

$$\langle \Psi | \Psi \rangle \ < \infty; \tag{7}$$

this simply states that Ψ must belong to the Hilbert space $\mathcal{H}$. Note that these arguments do not assume H to be essentially self-adjoint, which is often not easy to show. It suffices to ensure that a self-adjoint extension for H *exists*, for which one has simple powerful criteria [17].

In what follows, we apply this procedure within the context of EYM theory.

3 Existence of odd–parity negative modes for EYM black holes

All known essentially non-Abelian EYM black hole solutions [2], [6] can be obtained within the context of the $SU(2) \times U(1)$ EYM theory. The corresponding action is

$$S = -\frac{1}{16\pi G}\int R\sqrt{-g}d^4x - \int \left(\frac{1}{2e^2}tr F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right)\sqrt{-g}d^4x, \tag{8}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ is the matrix valued gauge field tensor, e is the gauge coupling constant, $A_\mu = A^a_\mu \tau^a/2$, and τ^a $(a = 1, 2, 3)$ are the Pauli matrices; $\mathcal{F}_{\mu\nu}$ is the $U(1)$ field strength.

In the spherically symmetric case, the spacetime metric is chosen to be

$$ds^2 = l_e^2\left((1 - 2m/r)\sigma^2 dt^2 - \frac{dr^2}{1 - 2m/r} - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right). \tag{9}$$

Here $l_e = \sqrt{4\pi}l_{pl}/e$ is the only dimensional quantity in the problem (l_{pl} being the Planck length); the functions m and σ depend on t and r. The $U(1)$ part of the gauge field is chosen to be of the dyon type, $e\mathcal{F} = (q_e/r^2)dt \wedge dr + q_m \sin\vartheta \; d\vartheta \wedge d\varphi$, satisfying the Maxwell equations for any constant q_e and q_m.

The spherically symmetric $SU(2)$ YM field can be parameterized by

$$eA = W_0\hat{L}_1\, dt + W_1\hat{L}_1\, dr + \{p_2\,\hat{L}_2 - (1-p_1)\,\hat{L}_3\}\, d\vartheta + \{(1-p_1)\,\hat{L}_2 + p_2\,\hat{L}_3\}\sin\vartheta\, d\varphi, \tag{10}$$

where W_0, W_1, p_1, p_2 are functions of t and r, $\hat{L}_1 = n^a\tau^a/2$, $\hat{L}_2 = \partial_\vartheta\hat{L}_1$, $\sin\vartheta\hat{L}_3 = \partial_\varphi\hat{L}_1$, and $n^a = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$. The gauge transformation

$$A \to \; UAU^{-1} + iUdU^{-1}, \quad \text{with} \;\; U = \exp(i\Omega(t,r)\hat{L}_1), \tag{11}$$

preserves the form of the field (10), altering the functions W_0, W_1, p_1, p_2 as

$$W_0 \to W_0 + \dot{\Omega}, \quad W_1 \to W_1 + \Omega', \quad p_\pm = p_1 \pm ip_2 \to \exp(\pm i\Omega)p_\pm; \tag{12}$$

here dot and prime denote differentiation with respect to t and r, respectively.

It is convenient to introduce the complex variable $f = p_1 + ip_2$ and its covariant derivative $D_\mu f = (\partial_\mu - iW_\mu)f$, as well as the $(1+1)$-dimensional field strength $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $(\mu, \nu = 0, 1)$. The full system of the EYM equations then reads

$$\partial_\mu(r^2\sigma W^{\mu\nu}) - 2\sigma \; Im \; (fD^\nu f)^* = 0, \tag{13}$$

$$D_\mu\sigma D^\mu f - \frac{\sigma}{r^2}(|f|^2 - 1)f = 0, \tag{14}$$

$$m' = -\frac{r^2}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{N\sigma^2}|D_0 f|^2 + N|D_1 f|^2 + \frac{1}{2r^2}(|f|^2 - 1)^2 + \frac{q^2}{2r^2}, \tag{15}$$

$$\dot{m} = 2N \; Re \; D_0 f(D_1 f)^*, \tag{16}$$

$$(\ln\sigma)' = \frac{2}{r}\left(\frac{1}{N^2\sigma^2}|D_0 f|^2 + |D_1 f|^2\right), \tag{17}$$

where $N = 1 - 2m/r$, $q^2 = q_e^2 + q_m^2$, and asterisk denotes complex conjugation.

Both for $q = 0$ (the $SU(2)$ case) [2], and $q \neq 0$ [6], these equations are known to possess static non-Abelian black hole solutions [2] labeled by a pair (n, r_H), where n is an integer, and $r_H \geq |q|$ is the black hole radius. For these solutions, $W_0 = W_1 = p_2 = 0$; the function $f = Re\, f = p_1$ has n nodes in the domain $r_H < r < \infty$ and tends asymptotically to ± 1 always remaining in the stripe $-1 < f(r) < 1$. The metric functions N and σ increase monotonically from $N(r_H) = 0$ to $N(\infty) = 1$ and from $0 < \sigma(r_H) < 1$ to $\sigma(\infty) = 1$, respectively. Asymptotically, the metric is of the Reissner-Nordström (RN) form with charge q.

It is worth noting that $SU(2) \times U(1)$ group may also arise as a subgroup of a larger gauge group [6]. The basic EYM equations remain of course the same in this case, and the only difference may appear in the quantization condition for the Abelian magnetic charge q_m.

Consider small perturbations of a given black hole solution

$$m \to m + \delta m, \quad \sigma \to \sigma + \delta \sigma, \quad f \to f + \delta f, \quad W_0 = W_1 = 0 \to \delta W_0, \ \delta W_1. \qquad (18)$$

The linearization of the field equation reveals that the even–parity perturbations, $(\delta m, \delta \sigma, \delta p_1)$, and the odd–parity perturbations, $(\delta W_0, \delta W_1, \delta p_2)$, decouple, and therefore are independent — this is because the background solutions are invariant under parity. Notice that the infinitesimal gauge transformation (11) does not alter the even-parity perturbations, while the odd-parity ones change as

$$\delta W_0 \to \delta W_0 + \dot{\Omega}, \quad \delta W_1 \to \delta W_1 + \Omega', \quad \delta p_2 \to \delta p_2 + \Omega f. \qquad (19)$$

For the *even-parity* modes, the perturbation equations for $(\delta m, \delta \sigma, \delta p_1)$ can be reduced to a single Schrödinger-type equation. In the $q = 0$ case, this equation was analyzed numerically by Straumann and Zhou [3] who found exactly n bound states for any background (n, r_H) black hole solution. Here we will concentrate on the *odd–parity* sector, and our results do not depend on q.

For the odd-parity fluctuations, the metric remains unperturbed [12], thus the perturbation equations can easily be obtained via expanding the Yang-Mills equations (13), (14) with respect to δW_0, δW_1, and δp_2 alone. We use the gauge freedom (19) to ensure the condition $\delta W_0 = 0$, and specify the time dependence as $\delta W_1 = \sqrt{2/(r^2 N)}\, \alpha(r) \exp(-i\omega t)$, $\delta p_2 = \xi(r) \exp(-i\omega t)$. The resulting equations can be represented as the eigenvalue problem

$$H\Psi = \omega^2 \Psi, \qquad (20)$$

where $\Psi = \begin{pmatrix} \alpha \\ \xi \end{pmatrix}$, and the Hamiltonian is given by

$$H = \sigma N \begin{pmatrix} 2\sigma f^2/r^2 & \sigma\sqrt{2N/r^2}(f' - if\hat{p}) \\ (f' + i\hat{p}f)\sigma\sqrt{2N/r^2} & \hat{p}\sigma N\hat{p} + \sigma(f^2 - 1)/r^2 \end{pmatrix}. \qquad (21)$$

The quantities f, σ, N refer to the background static black hole solution, and $\hat{p} = -id/dr$. There exists also an additional equation due to the Gauss constraint (Eq.(13) with $\nu = 0$)

$$\omega \left(\sqrt{\frac{2r^2}{\sigma^2 N}} \alpha \right)' = \omega \frac{2}{\sigma N} f\xi. \qquad (22)$$

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One can see however that, as long as $\omega \neq 0$, this equation is a differential consequence of (20), (21). We observe therefore that any solutions to the eigenvalue problem (20) (21) with $\omega \neq 0$ automatically satisfy the Gauss constraint, that is, they have correct initial values.

We introduce the tortoise coordinate $r_* \in \mathrm{R}$, such that $dr_* = dr/(\sigma N)$, and define the inner product as

$$\langle \Psi_1 | \Psi_2 \rangle = \int_{-\infty}^{\infty} \Psi_1^\dagger \Psi_2 \, dr_* = \int_{-\infty}^{\infty} (\alpha_1^* \alpha_2 + \xi_1^* \xi_2) dr_*. \tag{23}$$

The Hilbert space can then be chosen to be $\mathcal{H} = \mathrm{L}^2(\mathrm{R}, dr_*) \oplus \mathrm{L}^2(\mathrm{R}, dr_*)$. Consider the dense set in $\mathcal{H}$ consisting of twice continuously differentiable functions with compact support. H maps this set into $\mathcal{H}$ and it is symmetric on this set with respect to the inner product (23). We can therefore specify this set as the domain of H, $\mathcal{D}(H) = \mathrm{C}_0^2(\mathrm{R}) \times \mathrm{C}_0^2(\mathrm{R}) \subset \mathcal{H}$ [17], (in addition, H can be shown to be bounded from below on $\mathcal{D}(H)$).

Notice that H, being real, commutes with complex conjugation. A theorem by Von Neumann [17] then ensures the existence of self-adjoint extensions for H. Thus, according to the general procedure outlined above, to demonstrate instability it is sufficient to find a function $\Psi \in \mathcal{D}(H)$ satisfying the inequality (4).

Consider the set of real functions $\{h_k(r_*)\} \subset \mathrm{C}_0^2(\mathrm{R})$, where $h_k(r_*) = h(r_*/k)$, $k \geq 1$, and $h(r_*)$ is defined as follows: $h(r_*) = h(-r_*)$, $h = 1$ when $r_* \in [0, a]$, $-D \leq h_*' < 0$ for $r_* \in [a, a+1]$, and $h = 0$ for $r_* > a + 1$. Here a, D are positive constants, prime and asterisk denote differentiation with respect to r_*, $h_*' \equiv dh/dr_* = \sigma N h'$.

Using $\{h_k\}$, construct the set $\{\Psi_k\} \subset \mathcal{D}(H)$ specified by

$$\alpha_k = \sqrt{2Nr^2} \sigma f_*' \frac{(f^2 - 1)}{r^2} h_k \equiv \alpha_0 h_k, \quad \xi_k = (f_*')^2 h_k \equiv \xi_0 h_k, \tag{24}$$

(notice that the background solutions (f, f', N, σ) are at least twice differentiable [18]). Integration by parts allows us to represent the expectation value $\langle \Psi_k | H | \Psi_k \rangle$ in the form

$$\langle H \rangle = \int_{-\infty}^{\infty} \left\{ \left((\xi_k)_*' - f\sigma N \sqrt{\frac{2}{Nr^2}} \alpha_k \right)^2 + 2\sigma N f_*' \sqrt{\frac{2}{Nr^2}} \alpha_k \xi_k + \frac{\sigma^2 N}{r^2} (f^2 - 1) \xi_k^2 \right\} dr_*. \tag{25}$$

Substituting (24) into (25) and taking into account the background Yang-Mills equation

$$f_{**}'' = \frac{\sigma^2 N}{r^2} f(f^2 - 1) \tag{26}$$

we obtain

$$\langle \Psi_k | H | \Psi_k \rangle = 5 \int_{-\infty}^{\infty} dr_* \, \xi_0^2 N \sigma^2 \frac{(f^2 - 1)}{r^2} +$$

$$\int_{-\infty}^{\infty} dr_* \xi_0^2 \left\{ (h_k)_*'^2 + 5 \frac{N\sigma^2}{r^2} (1 - f^2)(1 - h_k^2) \right\} \equiv I + I(k). \tag{27}$$

In this equation, the k–indepedent first term I is finite and manifestly negative (recall that $|f| < 1$, in addition, $f_*' \sim N \to 0$ as $r_* \to -\infty$, $f_*' \sim 1/r_*^2$ as $r_* \to \infty$, so the integral exists). Taking into account the properties of the smoothening functions h_k specified above, one can see that the integral $I(k)$ converges uniformly and tends to zero

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as k increases. We therefore conclude that, for large k, the contribution of $I(k)$ into (27) is negligible, thus the whole expression turns out to be negative. This shows that Ψ_k with sufficiently large k are such functions from $\mathcal{D}(H)$ that $\langle \Psi_k | H | \Psi_k \rangle < 0$, which completes the proof of the existence of the odd-parity negative modes. Physically, these Ψ correspond to such admissible time-symmetric initial data which give rise to a growing instability [19].

Our analysis is valid for any $r_H > |q|$. For the uncharged $SU(2)$ solutions, the limit $r_H \to 0$ is known to relate to the regular case. Then $N(r) > 0$ for any $r \geq 0$, so r_* runs over semi-axis, and the Hilbert space is therefore $\mathcal{H} = \mathrm{L}^2(\mathrm{R}_+, dr_*) \oplus \mathrm{L}^2(\mathrm{R}_+, dr_*)$. One can see that the analysis presented above is equally valid in this case.

Finally, for the sake of completeness, we analyse stability of the Abelian magnetic $U(1)$ black holes. For these solutions, the dynamics of perturbations is still governed by Eqs.(20), (21), where the parameters of background solutions are $W_0 = W_1 = f \equiv 0$, $\sigma \equiv 1$, $N = 1 - 2M/r + 1/r^2 \equiv (r - r_+)(r - r_-)/r^2$, M being the ADM mass. We specify $\mathcal{D}(H)$ as before and choose the set $\{\Psi_k\} \subset \mathcal{D}(H)$ parameterized by $\alpha_k = 0$, $\xi_k = (r - r_+)/r^2 h_k$. Repeating the above procedure, one arrives at

$$\langle \Psi_k | H | \Psi_k \rangle = \int_{-\infty}^{\infty} dr_* \left((\xi_k)_*'^2 - \frac{N}{r^2} \xi_k^2 \right) = -\frac{7r_+ + 5r_-}{420 r_+^4} + I(k), \qquad (28)$$

where $I(k) \to 0$ with growing k. This shows that Abelian EYM black holes are unstable with respect to non–Abelian fluctuations (see also [20]).

4 Discussion

Our analysis reveals the existence of at least one *odd–parity* negative mode for all known non-Abelian EYM black hole solutions, indicating in particular that all of them are unstable. For the uncharged $SU(2)$ solutions, taking into account the results of the analysis by Straumann and Zhou [3], we therefore conclude that each (n, r_H) black hole has at least $n + 1$ unstable modes – n in the even-parity sector and at least one odd-parity negative mode. Our analysis is equally valid in the regular case, where such an odd-parity mode has precisely the same meaning as negative mode of the electroweak sphaleron solution [7], [8]. In this sense, this mode is fairly interesting and, from the physical point of view, quite typical. In the regular case, it had been precisely the existence of this mode which allowed us to suggest a sphaleron interpretation for the BK solutions [10]. On the other hand, the existence of the *even-parity* sphaleron negative modes is a rather peculiar phenomenon which is present in the EYM theory [11].

For black holes, as has already been mentioned [15], the sphaleron interpretation is not so transparent. By definition, a sphaleron is a static solution "sitting" at the top of a potential barrier separating topologically distinct YM vacua, whereas in the black hole case there are no pure vacuum states because of the finite temperature associated with the event horizon (except for the extreme case [6], [21]). The existence of the odd–parity negative modes in the black hole case shows, however, that the structure of the energy surface in the vicinity of the EYM black hole solutions in function space is similar to that for the regular BK objects. This suggests one to think of them as "black holes inside EYM sphalerons". An interesting open issue is the *exact* number of the odd–parity modes both for regular and black hole EYM solutions. Investigation of this requires a further numerical work.

Acknowledgments

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